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RESEARCH IN PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS.(U)
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RESEARCH IN PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS.

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FINAL REPORT

by

ALFRED S. CARASSO, Principal Investigator

AD A105859

August 30, 1981

U.S. ARMY RESEARCH OFFICE

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Grant # DAAG29-76-G-0153, DAAG29-78-G-0091

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(18) ARO, ARO

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A105859	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) RESEARCH IN PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS. ~~FINAL REPORT.~~		5. TYPE OF REPORT & PERIOD COVERED FINAL REPORT, March 1976 – June 1981.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Alfred S. CARASSO, Principal Investigator		8. CONTRACT OR GRANT NUMBER(s) DAAG29-76-G-0153, DAAG29-78-G-0091
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mathematics and Statistics University of New Mexico, Albuquerque, NM, 87131		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE Aug 1981
		13. NUMBER OF PAGES 5
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE NA

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

(11) 13355.5-M, 15168.1-M

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Final report, digital image restoration, plasma compression by magnetic pistons, heat transfer in gun barrels, ill-posed initial value problems, snow-plow model, inverse problems for parabolic equations.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This final report summarizes work accomplished during the period March 1 1976 through June 30 1981. The research deals with the analysis and efficient computation of several ill-posed initial value problems, linear and nonlinear. These problems originate in digital image restoration, plasma compression by magnetic pistons, and heat transfer in gun barrels. The analyses and computational algorithms developed are closely tied to the contextual origin of each problem studied.

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Introduction.

This final report summarizes work accomplished under Grant # DAAG29-76-G-0153 and its continuation, Grant # DAAG29-78-G-0091, during the period March 1, 1976 through June 30, 1981. During this period, seven research manuscripts were completed and submitted for publication, with copies of each manuscript transmitted to ARO. These are listed below by title under items 1 through 7, together with the current status of each paper relative to publication in the literature. Below each title, a summary is appended describing the problem studied in each case, and the principal results obtained.

The research carried out deals with the analysis and efficient computation of several ill-posed initial value problems for partial differential equations, linear and nonlinear. These problems originate in various contexts, such as digital image processing and enhancement, plasma compression by magnetic pistons, and heat transfer in gun barrels. Accordingly, the analyses carried out, and the computational algorithms and examples which are developed, are closely tied to the contextual origin of each problem.

Scientific Personnel Supported by this Project.

Alfred Carasso, Principal Investigator.

No advanced degrees were awarded.

1. Digital removal of random media image degradations by solving the diffusion equation backwards in time. SIAM J. Numer. Anal. 15, (1978), pp. 344-367 by Alfred Carasso, James Sanderson and James Hyman.

This lengthy research article gives a thorough analysis of the image restoration problem for Gaussian-like point spread functions, together with examples of actual degraded images and their improvement by this technique. In the first part of the article, the restoration problem is reformulated as an initial value problem for a backwards parabolic or a pseudo-parabolic partial differential equation in two space dimensions. This approach leads to rigorous bounds on the reliability of the restoration, as a function of the noise variance, without any assumptions on the spectral characteristics of either signal or noise. Such reliability estimates were not previously known in the image processing literature. In the second part of the paper, the backward beam equation approach is coupled with Fast Fourier Transform algorithms to create a powerful new algorithm for computer enhancement of such images. The algorithm can be applied in several ways to arrive at credible restorations even when the noise level in the degraded image is not precisely known. This feature is demonstrated by applying the technique to an actual turbulence degraded image with excellent results. Finally, the performance of the algorithm is studied, on simulated blurred images, at various levels of signal to noise ratios.

2. A system of nonlinear partial differential equations describing cylindrical plasma collapse. Report LA-6824, (1978), Los Alamos Scientific Laboratory, Los Alamos, NM 87545, by Alfred Carasso and B.R. Suydam.

The underlying physical problem considered here is the compression of a fully ionized plasma by a magnetic piston. The problem originated at Los Alamos and was brought to our attention by B.R. Suydam of the Mathematical Modeling and Analysis Group (T-7) at Los Alamos. We consider the 'snow-plow' model for cylindrical plasma collapse in the case of a specified constant driving pressure. Mathematically, this consists of a coupled system of five nonlinear partial differential equations in two independent variables, one of which is the time. The initial value problem for similar linear systems is improperly posed. We prove that this is not the case here, by direct construction of the unique solution, explicitly in terms of the initial data. The solution is found to develop non-physical singularities after a certain positive time, and ceases to relate to the underlying physical problem. The theory provides an upper bound on the time interval during which the snow-plow model is realistic, in terms of the initial data. Several computational examples are given to illustrate the pathologies exhibited by the solution.

3. On a simplified model for toroidal plasma compression. Unpublished Manuscript, by Alfred Carasso and B.R. Suydam.

The preceding analysis in cylindrical geometry discussed above, is here generalized so as to apply to the general 3-D problem in toroidal geometry. Assuming a constant positive driving pressure, and an initial plasma-vacuum interface homeomorphic to a torus, the nonlinear initial value problem can again be solved explicitly in terms of the initial data. An elegant new treatment shows that the solution at time t now depends on the Gaussian and Mean Curvatures of the initial interface. The solution always develops a non-physical singularity at some positive time T, whose value is inversely proportional to the maximum curvature of the initial interface. This analysis includes the preceding analysis in cylindrical geometry as a special case.

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4. A singular initial value problem in plasma collapse theory. Unpublished Manuscript. by Alfred Carasso and B.R. Suydam.

This research manuscript is a sequel to items 2 and 3 above, where a general treatment was possible under the assumption of a constant positive driving pressure. The more general case where the driving pressure is a specified function of time, is considerably more delicate, and appears fraught with serious pathologies. In the present paper we consider the snow plow model with a time dependent driving pressure, in the simple case where an initially circular interface remains circular during the compression. In this case, the snow plow equations reduce to a first order system of two nonlinear ordinary differential equations. This system has two singular points, and does not lend intself to the usual existence-uniqueness theorems. Rather, using new approximative techniques, together with the Ascoli-Arzela and Lebesgue dominated convergence theorems, we establish global existence and uniqueness results under certain hypotheses regarding the behavior of the driving pressure at $t = 0$. We also show that the radius of the interface shrinks to zero in a finite time T_c, and that the solution is not relevant to the physical problem for values of t greater than T_c.

5. Inverse Problems in Dissipative Evolution Equations and Applications. In 'Proceedings of the International Symposium on Ill-Posed Problems' Z. Nashed, Editor, University of Delaware, October 1979, D. Reidel Publishing Co., Hingham, Mass. (in press), by Alfred Carasso.

The first part of this manuscript deals with a preliminary report on the 'inverse heat conduction problem' via the sideways heat equation. This research is directly relevant to the problem of gun tube erosion. For the case of the one dimensional heat equation, we show how to solve the inverse problem by means of A Tikhonov regularization procedure in a suitable norm. This method provides an approximation to the temperature history arbitrarily close to the wall. The approximation can be generated numerically by solving an appropriate initial value problem for the heat equation ran sideways. Error bounds of logarithmic convexity type are obtained, for the L_2 norm of the approximate solution. The treatment of this important problem via the sideways heat equation, as well as the error bounds obtained, are new. In the second part of this manuscript, we discuss the backwards problem for linear self adjoint parabolic equations with time dependent coefficients, and we survey the known results regarding stability under a prescribed bound. The theory of such backwards problem is illustrated with the example of digital restoration of Gaussian-blurred images. The contents of this manuscript were presented at the International Symposium on Ill-Posed Problems, at the University of Delaware in October, 1979, where the author was an invited speaker.

6. Determining surface temperatures from interior observations. Accepted for publication in SIAM J. Applied Math. Part A., by Alfred Carasso.

This research paper gives a thorough treatment of the inverse heat transfer problem in the one dimensional constant coefficient case. Error estimates are obtained for the regularized solution and its gradient. The regularized solution is generated numerically by solving an initial value problem for the sideways heat equation. The initial data for this problem is generated from recorded temperature histories at two thermocouple locations. Complete details are given regarding the numerical implementation of this scheme, and a numerical experiment, using hypothetical noisy data, is described. It is shown that temperature histories at the wall can be reconstructed with a pointwise accuracy of about 99%, with data typical of gun barrels. The last part of the manuscript discusses the extension to the nonlinear case of temperature-dependent thermal properties. An explicit scheme is written down and a prescription is given for appropriately filtering the solution at every step of the calculation. The filtering function is obtained from the related constant coefficient problem.

7. A stable marching scheme for an ill posed initial value problem. Accepted for publication in Mathematics of Computation, by Alfred Carasso.

This manuscript explores a new technique for solving ill posed initial value problems numerically. We study a marching scheme where the solution is appropriately filtered at every step of the calculation, using a filtering function constructed via a Tikhonov regularization of a related constant coefficient problem. The motivation for this study is provided by the nonlinear inverse heat transfer problem discussed above. Here, however, the ideas are applied to linear parabolic equations with variable coefficients and the time direction reversed. Such ill-posed problems are considerably more violent than the inverse heat transfer problem, with respect to noise amplification, and provide an ideal test of the robustness of this new algorithm. For variable coefficient problems whose coefficients are small perturbations away from constants, we derive error bounds which are close to best possible. Several numerical experiments are performed with this scheme, on problems with rapidly varying coefficients, and excellent results are obtained. The results of this manuscript appear to have important application in the calculation of many types of ill-posed initial value problems.

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